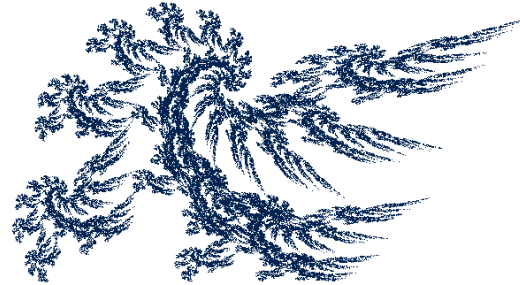
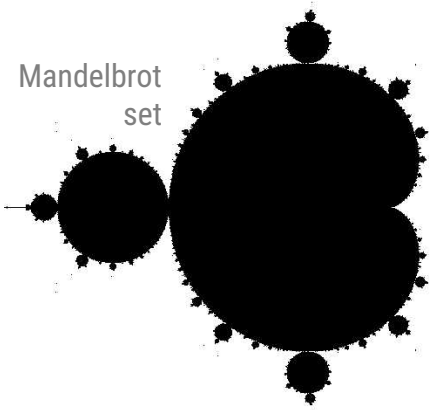


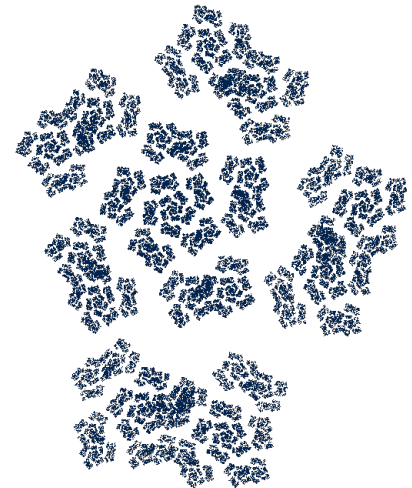
—  
Make your own fractal!

Mandelbrot  
set



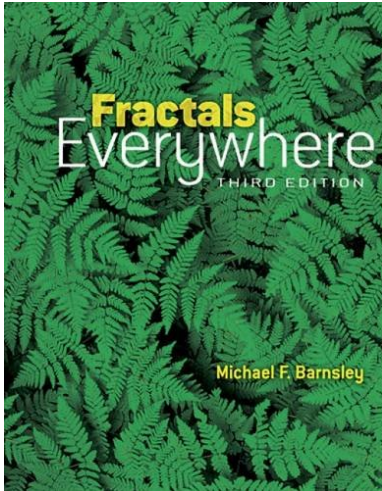
## What is a Fractal?

- Self-similar
- Nowhere differentiable
- Non-integer dimension





[nl.wikipedia.org/wiki/Romanesco\\_\(groente\)](https://nl.wikipedia.org/wiki/Romanesco_(groente))

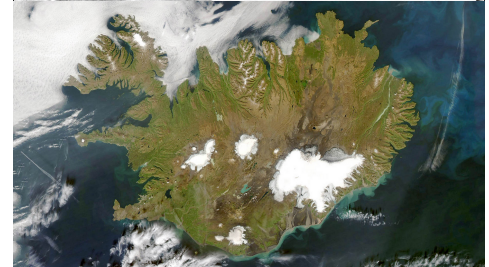
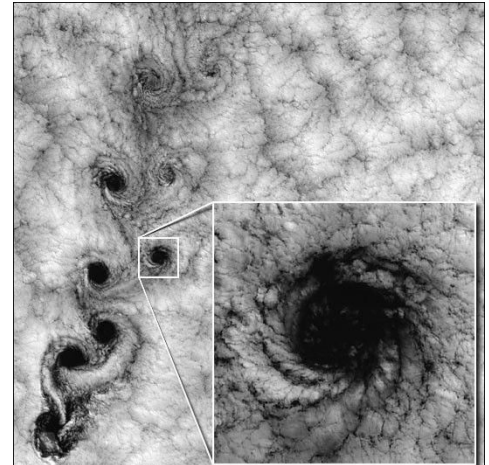


<https://m.media-amazon.com/images/I/61+q6AqRXhL.jpg>



[www.pflanzen-deutschland.de/Dryopteris\\_filix-mas.html](http://www.pflanzen-deutschland.de/Dryopteris_filix-mas.html)

[science.nasa.gov/earth/earth-observatory/landsat-7-reveals-large-scale-fractal-motion-of-clouds-625/](https://science.nasa.gov/earth/earth-observatory/landsat-7-reveals-large-scale-fractal-motion-of-clouds-625/)



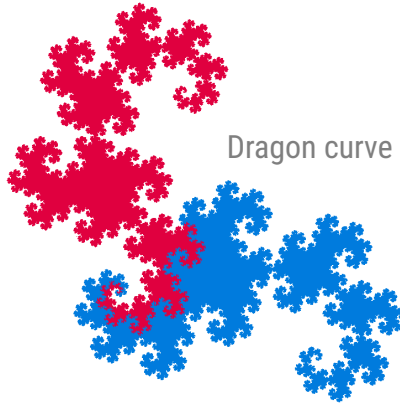
<https://eliasvetter.ch/Island/>

# Self-Similar Sets

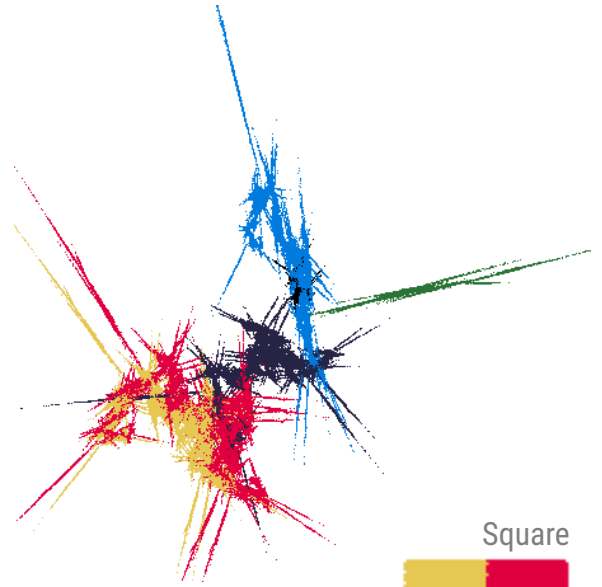
... consist of **finitely many** smaller copies of themselves



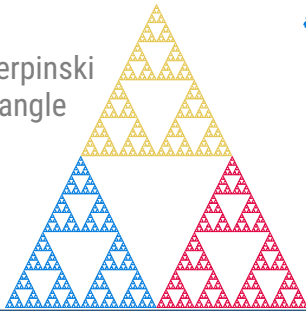
Barnsley fern



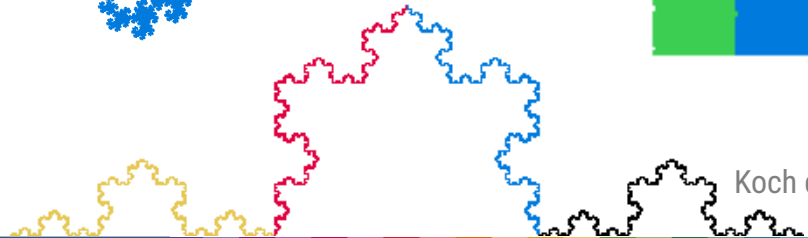
Dragon curve

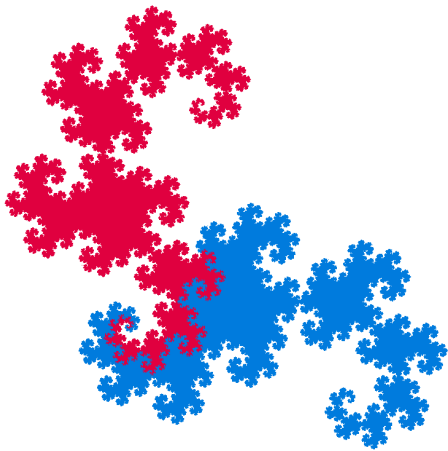


Sierpinski triangle



Koch curve

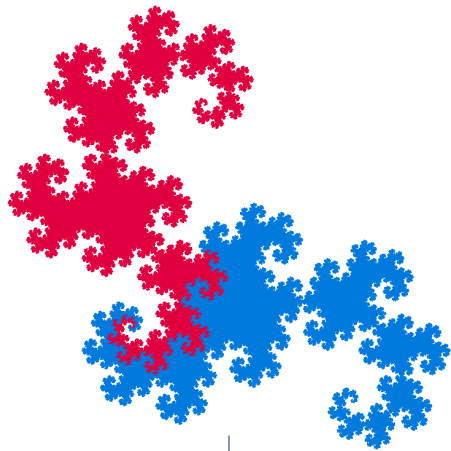




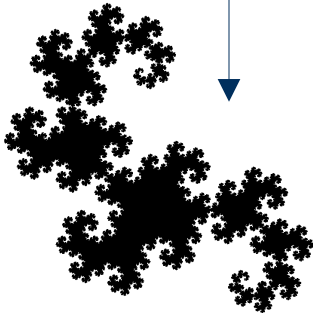
---

How do we describe a self-similar set?

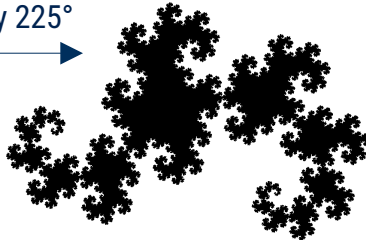
# How do we describe a self-similar set?



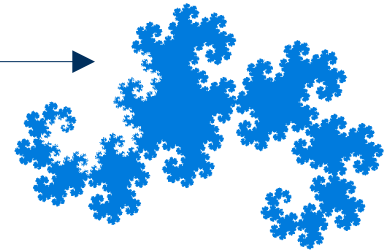
shrink by  $1/\sqrt{2}$



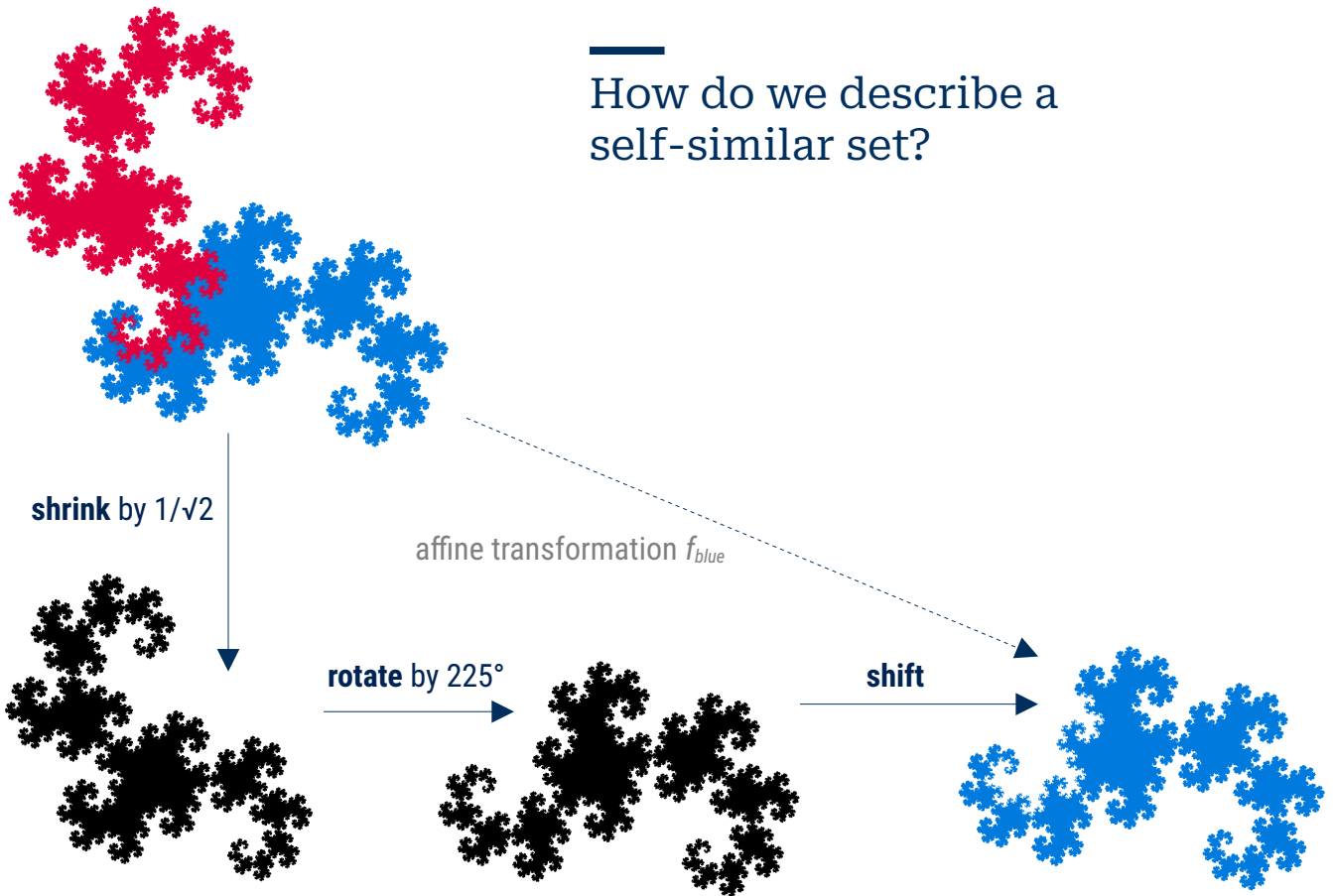
rotate by  $225^\circ$



shift



# How do we describe a self-similar set?

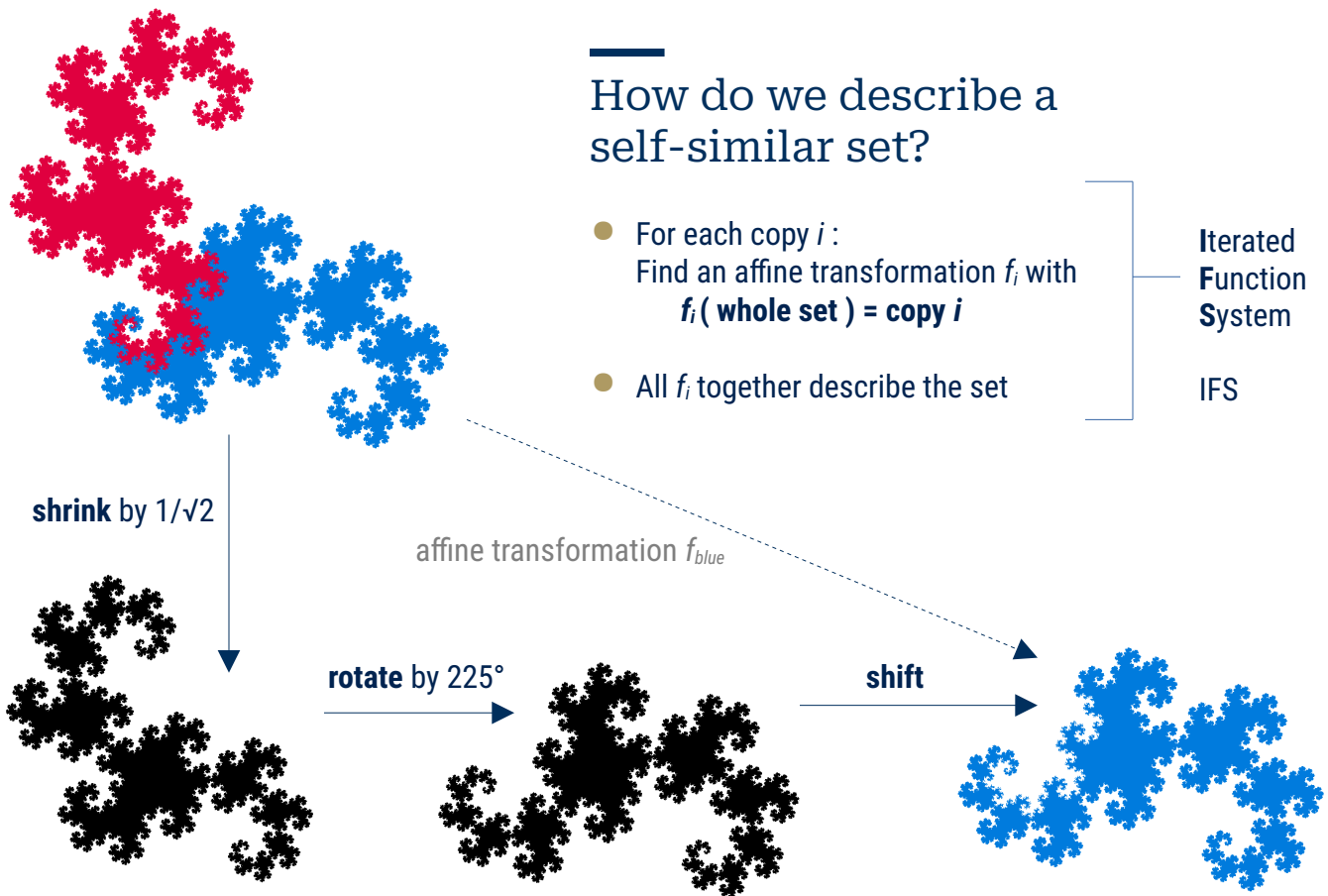


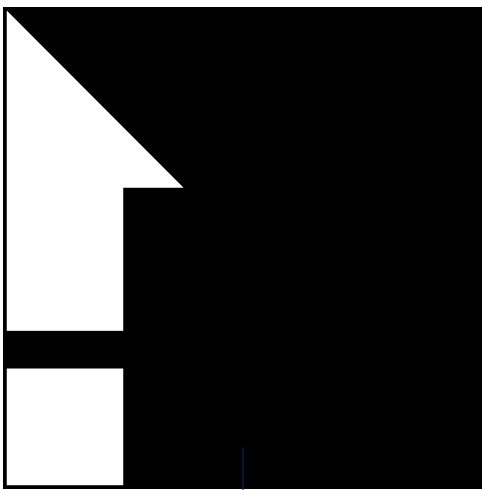
# How do we describe a self-similar set?

- For each copy  $i$  :  
Find an affine transformation  $f_i$  with  
 $f_i(\text{whole set}) = \text{copy } i$
- All  $f_i$  together describe the set

Iterated  
Function  
System

IFS





## How do we describe a self-similar set?

- For each copy  $i$  :  
Find an affine transformation  $f_i$  with  
 **$f_i(\text{whole set}) = \text{copy } i$**
- All  $f_i$  together describe the set

Iterated  
Function  
System

IFS

shrink by  $1/\sqrt{2}$

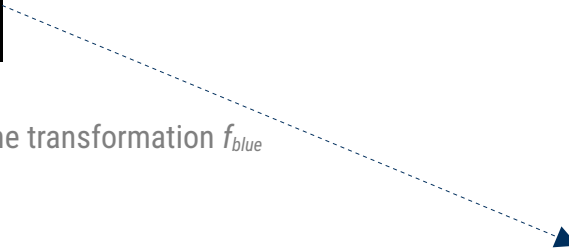


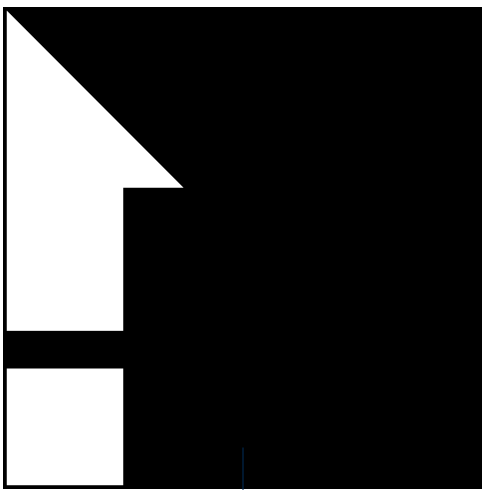
affine transformation  $f_{blue}$

rotate by  $225^\circ$



shift





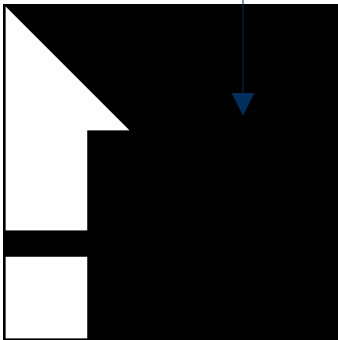
## How do we describe a self-similar set?

- For each copy  $i$  :  
Find an affine transformation  $f_i$  with  
 $f_i(\text{whole set}) = \text{copy } i$
- All  $f_i$  together describe the set

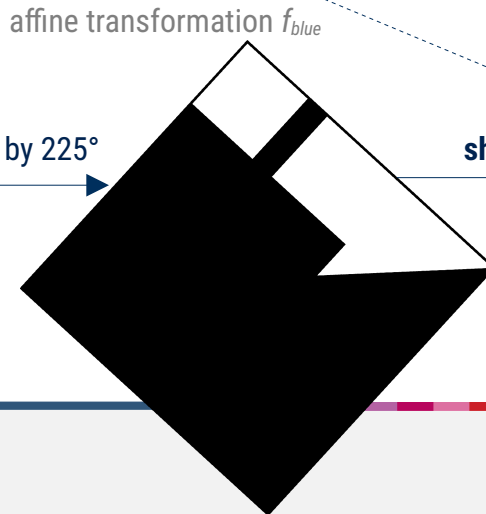
Iterated  
Function  
System

IFS

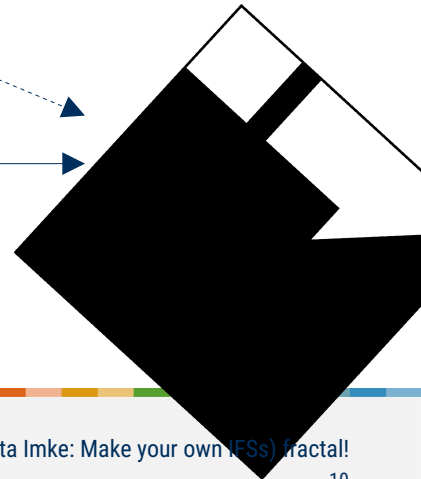
shrink by  $1/\sqrt{2}$



rotate by  $225^\circ$



shift

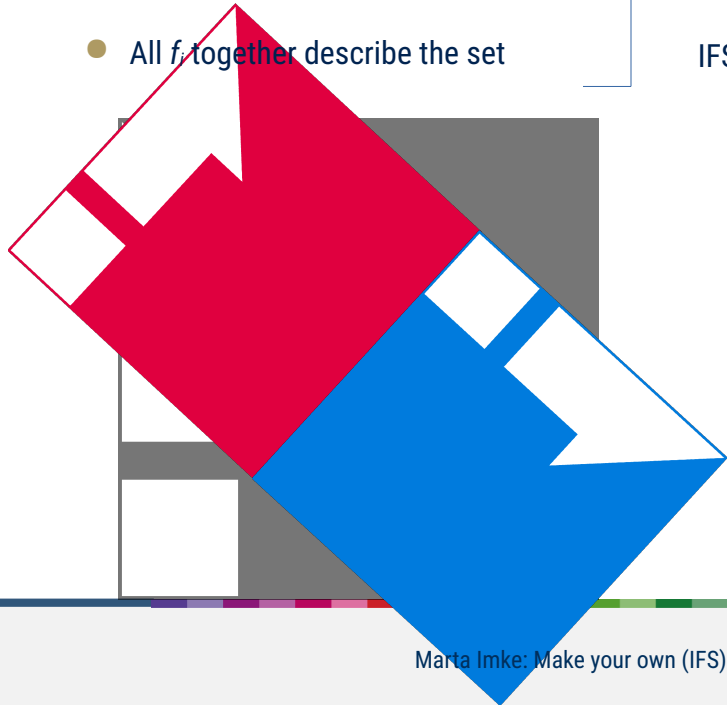
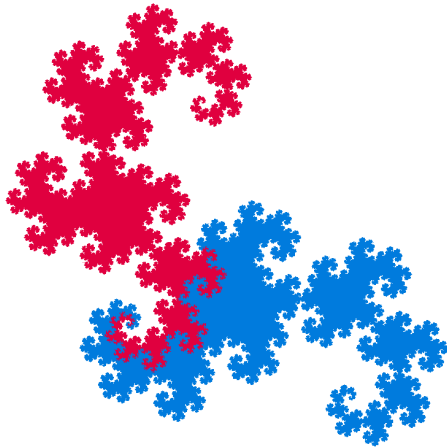


# How do we describe a self-similar set?

- For each copy  $i$  :  
Find an affine transformation  $f_i$  with  
 **$f_i(\text{whole set}) = \text{copy } i$**
- All  $f_i$  together describe the set

Iterated  
Function  
System

IFS



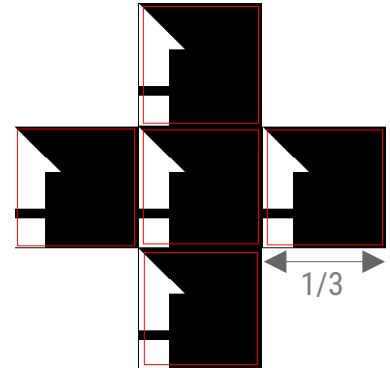
# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

**Copy and paste**

- 1) Start with any non-empty image

**Chaos Game**



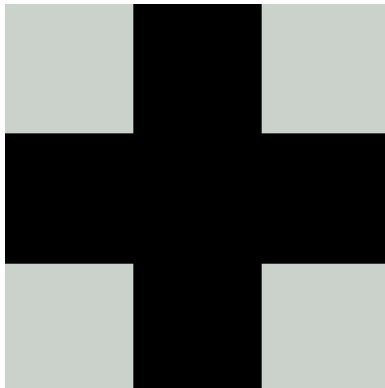
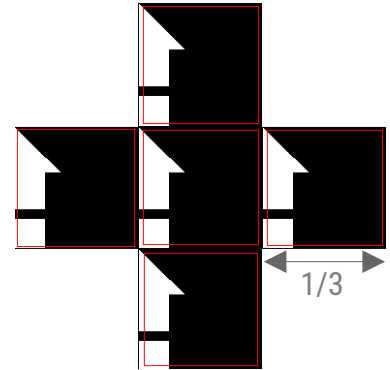
# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
transform the image

## Chaos Game

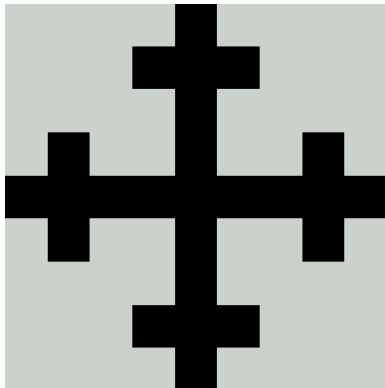


# How do I generate a self-similar set?

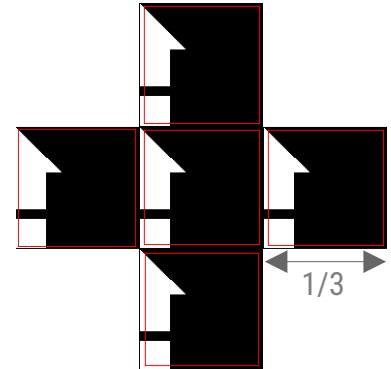
**Given:** affine transformations  $f_i$

## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game



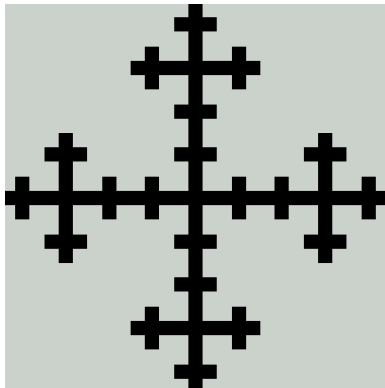
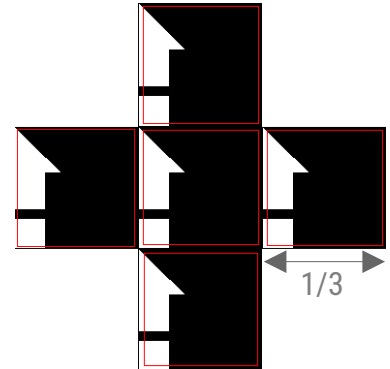
# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

**Copy and paste**

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat

**Chaos Game**



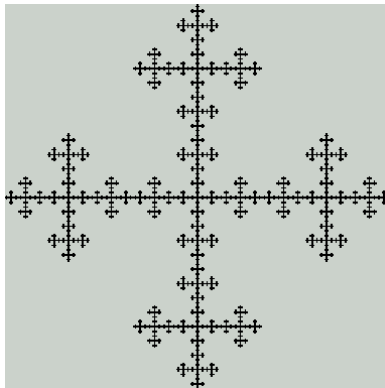
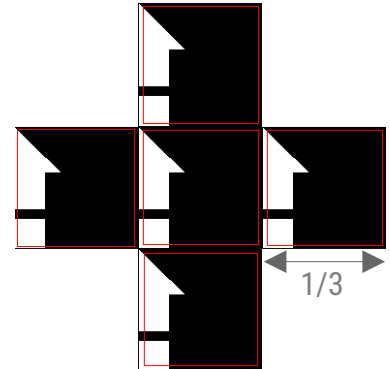
# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

**Copy and paste**

**Chaos Game**

- 1) Start with any non-empty image
- 2) For each  $i$ :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



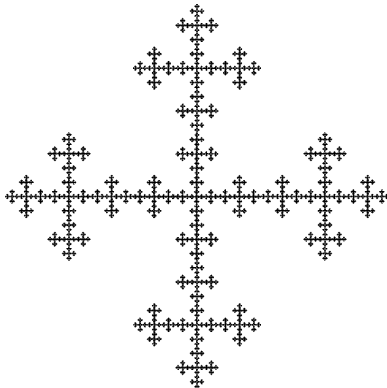
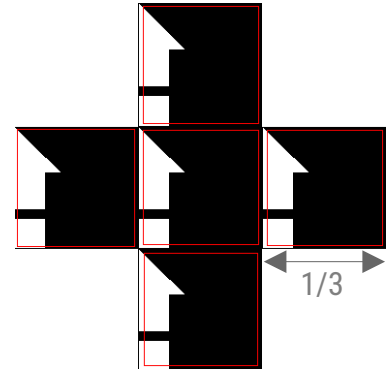
# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

**Copy and paste**

**Chaos Game**

- 1) Start with any non-empty image
- 2) For each  $i$ :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat

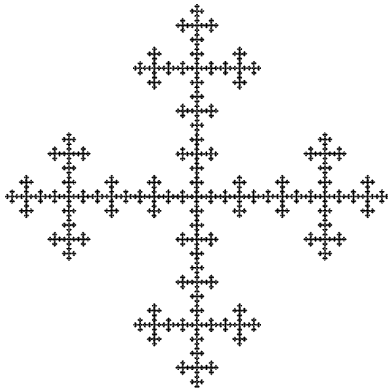


# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

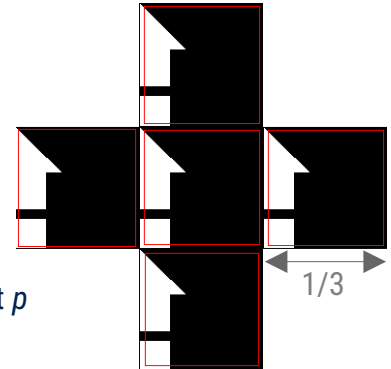
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$

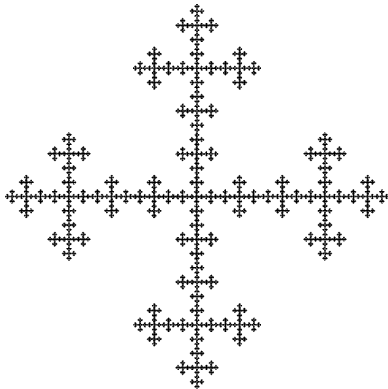


# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

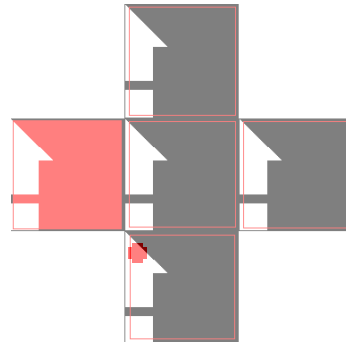
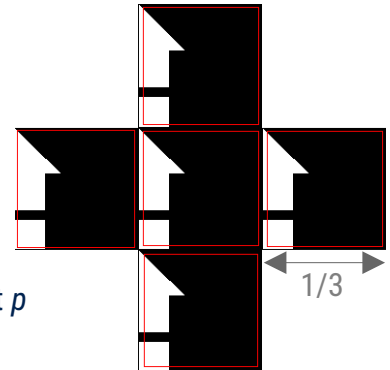
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$

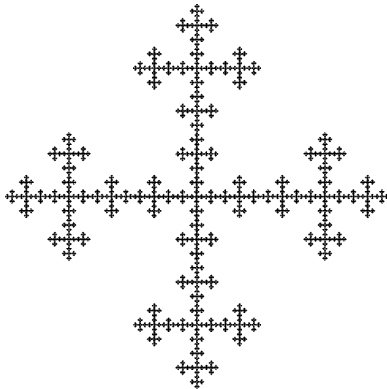


# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

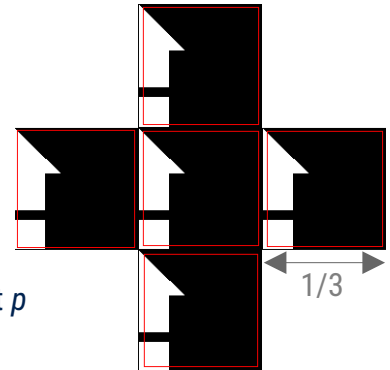
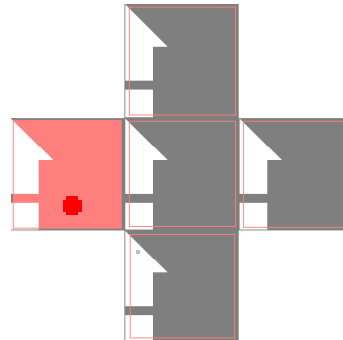
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$

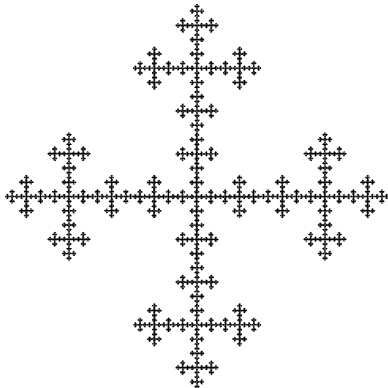


# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

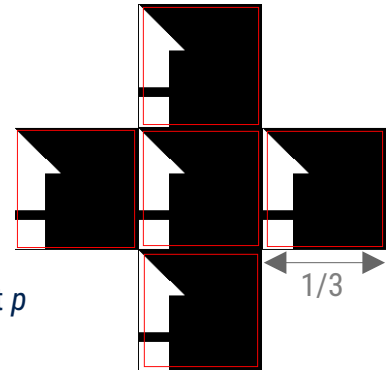
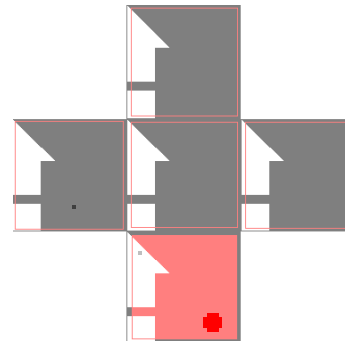
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$
- 5) Repeat



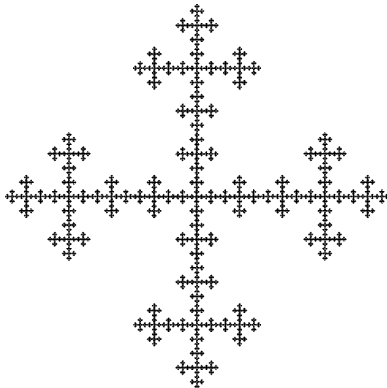
3 points

# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

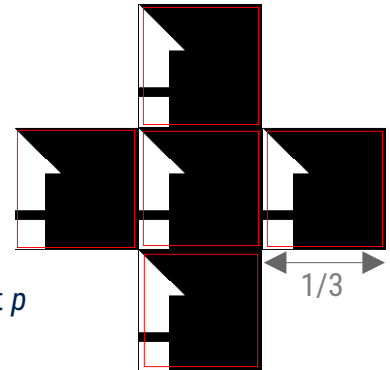
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$
- 5) Repeat



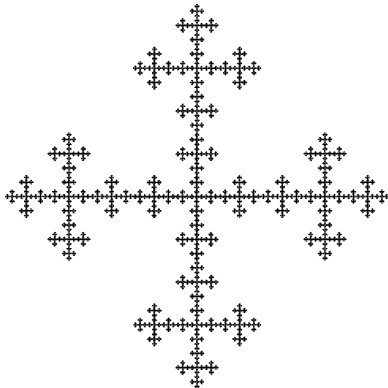
10 points

# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

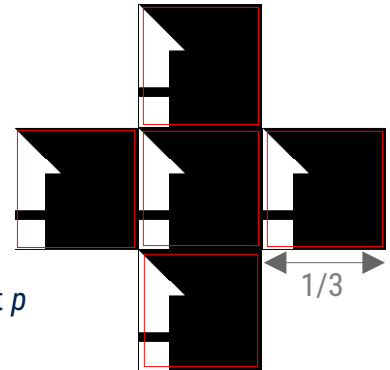
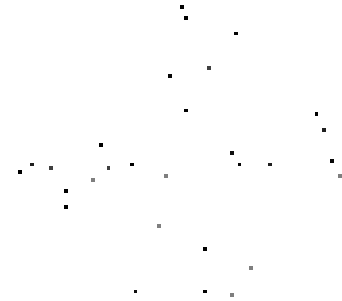
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$
- 5) Repeat



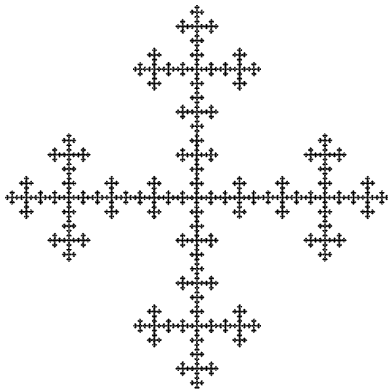
30 points

# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

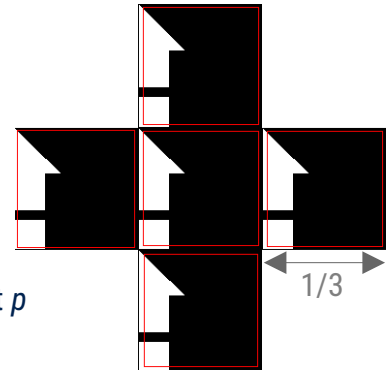
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$
- 5) Repeat



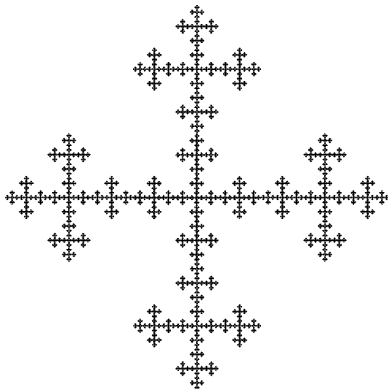
100 points

# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

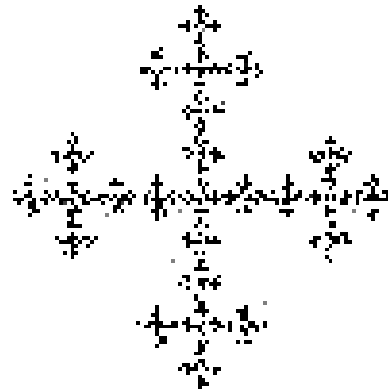
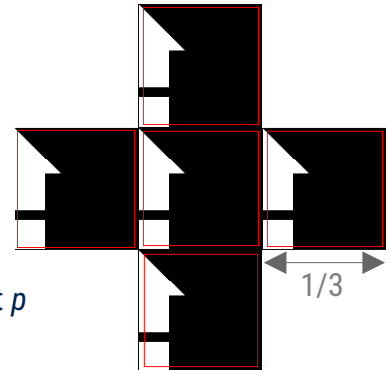
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$
- 5) Repeat



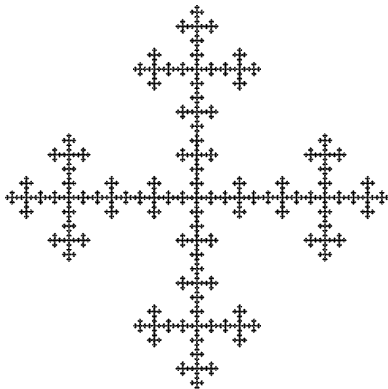
1,000 points

# How do I generate a self-similar set?

**Given:** affine transformations  $f_i$

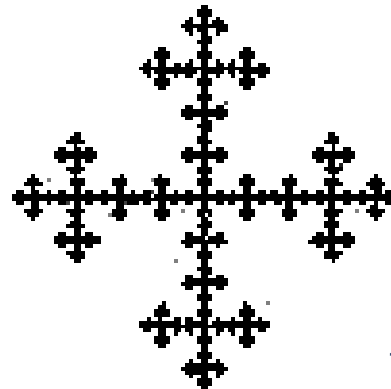
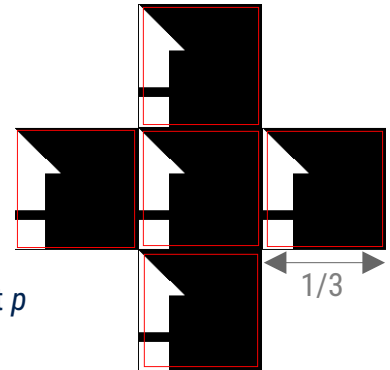
## Copy and paste

- 1) Start with any non-empty image
- 2) For each  $i$  :  
    transform the image
- 3) New image = union of transformed ones
- 4) Repeat



## Chaos Game

- 1) Start with any point  $p$
- 2) Draw  $p$
- 3) Select a random  $f_i$
- 4)  $new\_p = f_i(p)$
- 5) Repeat



10,000 points



See you(r fractal)  
on [fracmi.cc](http://fracmi.cc) !

